

1) (a)  $315^\circ$

Primary trig ratios

$$\sin 315^\circ = -\frac{1}{\sqrt{2}}, \text{ reciprocal} = -\sqrt{2}$$

b)  $120^\circ$

$$\text{prim } \cos 315^\circ = \frac{1}{\sqrt{2}}, \text{ reciprocal} = \sqrt{2}$$

$$\tan 315^\circ = -1, \text{ reciprocal} = -1$$

b)  $120^\circ$

Primary:

$$\sin 120^\circ = \frac{\sqrt{3}}{2}$$

reciprocal

$$= \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cos 120^\circ = -\frac{1}{2}$$

$$= -2$$

$$\tan 120^\circ = -\sqrt{3}$$

$$= -\frac{1}{\sqrt{3}} = -\frac{\sqrt{3}}{3}$$

(c)  $210^\circ$

Primary ratios

$$\sin 210^\circ = -\frac{1}{2}$$

Reciprocal

$$= -2$$

$$\cos 210^\circ = -\frac{\sqrt{3}}{2}$$

$$= -\frac{2}{\sqrt{3}} = -\frac{2\sqrt{3}}{3}$$

$$\tan 210^\circ = \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$= \sqrt{3}$$

d)  $60^\circ$

Primary

$$\sin 60^\circ = \frac{\sqrt{3}}{2}$$

Reciprocal

$$= \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cos 60^\circ = \frac{1}{2}$$

$$= 2$$

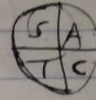
$$\tan 60^\circ = \sqrt{3}$$

$$= \frac{1}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

(2)

$$0 \leq \theta < 360$$

$$(a) \sin \theta = \frac{1}{2}$$



$$\theta = \sin^{-1} 0.5 = 30^\circ$$

$$\theta = 180 - 30 = 150^\circ$$

$$\theta = 30^\circ \text{ and } 150^\circ$$

$$(b) \cos \theta = -\frac{\sqrt{3}}{2}$$

$$\theta = \cos^{-1} -\frac{\sqrt{3}}{2} \text{ (negative)}$$

$$\theta = 30$$

$$\theta = 180 - 30 = 150^\circ \text{ (2nd quadrant)}$$

$$\theta = 180 + 30 = 210^\circ \text{ (3rd quadrant)}$$

$$\theta = 150^\circ \text{ and } 210^\circ$$

$$(c) \tan \theta = -1$$

$$\theta = \tan^{-1} -1 = -45^\circ$$

$$\theta = 180 - 45 = 135^\circ$$

$$\theta = 360 - 45 = 315^\circ$$

$$\theta = 135^\circ \text{ and } 315^\circ$$

$$(d) \cos \theta = \frac{1}{\sqrt{2}}$$

$$\theta = \cos^{-1} \frac{1}{\sqrt{2}} = 45^\circ$$

$$\theta = 360 - 45 = 315^\circ$$

$$\theta = 45^\circ \text{ and } 315^\circ$$

e)  $\tan \theta = \sqrt{3}$



$\theta = \tan^{-1} \sqrt{3} = 60^\circ$

$\theta = 180 + 60 = 240^\circ$

$\theta = \underline{60^\circ \text{ and } 240^\circ}$

(3) Identities

(a)  $1 - \sin^4 \theta = \cos^2 \theta (1 + \sin^2 \theta)$

L.H.S

$1 - \sin^4 \theta = (1 + \sin^2 \theta)(1 - \sin^2 \theta)$

but  $1 - \sin^2 \theta = \cos^2 \theta$ , from identity  $\cos^2 \theta + \sin^2 \theta = 1$

Therefore

$1 - \sin^4 \theta = \cos^2 \theta (1 + \sin^2 \theta)$

(b)  $\tan \theta \cos \theta = \sin \theta$

$\tan \theta = \frac{\sin \theta}{\cos \theta}$

from cross-multiplication,

$\tan \theta \cos \theta = \sin \theta$

(c)  $1 - \cos^2 x = \sin x \cos x \tan x$

From identity  $\cos^2 x + \sin^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\sin^2 x = \sin x \cdot \sin x$

$\tan x = \frac{\sin x}{\cos x} \Rightarrow \sin x = \tan x \cos x$

$1 - \cos^2 x = \sin x \cos x \tan x$

(d)  $\left(\frac{\cos \theta}{\sin \theta}\right) \left(\frac{1}{\cos^2 \theta}\right) (\sin^2 \theta) = \tan \theta$

Solution

L.H.S

$\left(\frac{\cos \theta}{\sin \theta}\right) \left(\frac{1}{\cos^2 \theta}\right) (\sin^2 \theta)$

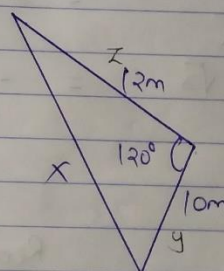
$= \left(\frac{1}{\tan \theta}\right) \left(\frac{\sin \theta}{\cos \theta} \cdot \frac{\sin \theta}{\cos \theta}\right)$

$= \frac{1}{\tan \theta} \cdot \tan \theta \cdot \tan \theta$

$= \underline{\tan \theta}$

(5)

(a)



Solution

Using cosine Rule

$x^2 = y^2 + z^2 - 2yz \cos x$

$= 10^2 + 12^2 - 2 \times 10 \times 12 \cos 120$

$= 100 + 144 - 240 \times -0.5$

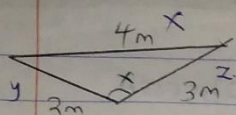
$= 244 - -120 = 364$

$x = \sqrt{364} = 19.0787 \text{ m}$

$x \approx \underline{\underline{19.08 \text{ m}}}$

5

(b)

Solution

From Cosine Rule,

$$x^2 = y^2 + z^2 - 2yz \cos X$$

$$\cos X = \frac{y^2 + z^2 - x^2}{2yz}$$

$$= \frac{2^2 + 3^2 - 4^2}{2 \times 3 \times 2} = -\frac{3}{12}$$

$$X = \cos^{-1}\left(-\frac{3}{12}\right) = 104.477^\circ$$

$$X \approx \underline{\underline{104.48^\circ}}$$

(6)

$$(a) \quad b = 3.0 \text{ cm}, \quad c = 5.5 \text{ cm}, \quad \angle B = 30^\circ$$

Solution

From Cosine Rule,

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$a^2 - 2ac \cos B = c^2 - b^2$$

$$a^2 - 2 \times 5.5a \cos 30^\circ = 9 - 30.25$$

$$a^2 - \frac{11}{2} \sqrt{3} a + \frac{8\sqrt{3}}{4} = 0$$

$$= \frac{\frac{11}{2} \sqrt{3} \pm \sqrt{\left(\frac{11}{2} \sqrt{3}\right)^2 - (4 \times \frac{8\sqrt{3}}{4})}}{2}$$

$$a_1 = 5.9621, \quad a_2 = 3.564$$

$$a_1 \approx 6.0 \text{ cm}, \quad a_2 \approx 3.6 \text{ cm}$$

$$a_1^2 = b^2 + c^2 - 2bc \cos A_1$$

$$\cos A_1 = \frac{b^2 + c^2 - a_1^2}{2bc}$$

$$= \frac{9 + 30.25 - 6.0^2}{2 \times 3 \times 5.5} = 0.09848$$

$$A_1 = \cos^{-1} 0.09848 = 84.35^\circ$$

$$A_1 \approx 84^\circ$$

$$\cos A_2 = \frac{b^2 + c^2 - a_2^2}{2bc}$$

$$= \frac{9 + 30.25 - 3.6^2}{2 \times 3 \times 5.5} = 0.7967$$

$$A_2 = \cos^{-1} 0.7967 = 37.187^\circ$$

$$A_2 \approx \underline{\underline{37^\circ}}$$

From Sine Rule,

$$\frac{a_1}{\sin A_1} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\sin C_1 = \frac{c \sin B}{b} = \frac{5.5 \sin 30^\circ}{3}$$

$$= 0.9167$$

$$C_1 = \sin^{-1}\left(\frac{1}{2}\right) = 66.44^\circ$$

$$\hat{=} \underline{\underline{66^\circ}}$$

$$\sin C_2 = \frac{c \sin A_2}{a_2} = \frac{5.5 \sin 37^\circ}{3.6}$$

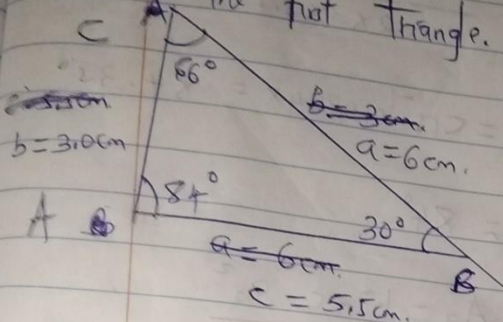
$$= 0.919$$

$$C_2 = 66^\circ$$

It is possible to draw a triangle which has the following

dimensions  
 $a = 6.0\text{cm}, b = 3.0\text{cm}, c = 5.5\text{cm}$   
 $\angle A = 66.84^\circ, \angle B = 30^\circ, \angle C = 66^\circ$

This is the first triangle.



$$b^2 = a^2 + c^2 - 2ac \cos B$$

(b)  $b = 12.2\text{cm}, c = 8.2\text{cm}$   
 Angle  $C = 34^\circ$

Solution

Using Cosine rule.

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$a^2 - 2ab \cos C = b^2 - c^2$$

$$a^2 - 2 \times 12.2 \cos 34^\circ = 12.2^2 - 8.2^2$$

$$a^2 - 20.239 = 81.6$$

$$a^2 - 20.239 - 81.6 = 0$$

$$a = \frac{20.239 \pm \sqrt{(20.239)^2 - (4 \times -81.6)}}{2}$$

$$a_1 = 23.67\text{cm}, a_2 = -3.446\text{cm}$$

$$a = 23.7\text{cm}$$

$$a^2 = b^2 + c^2 - 2bc \cos A$$

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

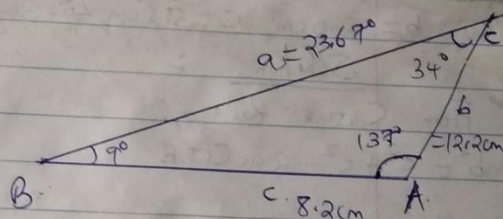
$$= \frac{12.2^2 + 8.2^2 - 23.7^2}{2 \times 12.2 \times 8.2}$$

$$= -1.727$$

$$A = 136.63^\circ \approx 137^\circ$$

It is possible to draw a triangle with sides

$a = 23.67\text{cm}, b = 12.2\text{cm}, c = 8.2\text{cm}$   
 angles  $\angle A = 137^\circ, \angle B = 9^\circ, \angle C = 34^\circ$



(c)  $a = 11.1\text{cm}, c = 5.2\text{cm}$   
 $\angle C = 33^\circ$

Solution

From Cosine Rule,

$$c^2 = a^2 + b^2 - 2ab \cos C$$

$$b^2 - 2ab \cos C = a^2 - c^2$$

$$b^2 - 2 \times 11.1 \cos 33^\circ = 11.1^2 - 5.2^2$$

$$b^2 - 18.62b - 96.17 = 0$$

$$b = \frac{18.26 \pm \sqrt{(18.26)^2 - (4 \times -96.17)}}{2}$$

$$b_1 = 22.5288 \text{ cm}, b_2 = -4.27 \text{ cm}$$

$$b \approx 22.5 \text{ cm}$$

$$b^2 = a^2 + c^2 - 2ac \cos B$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac} = \frac{11.1^2 + 5.2^2 - 22.5^2}{2 \times 11.1 \times 5.2}$$

$$= -3.08$$

$$B = 102.67^\circ \approx 103^\circ$$

From sine Rule,

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\sin A = \frac{a \sin C}{c} = \frac{11.1 \sin 33^\circ}{5.2} = 1.1626$$

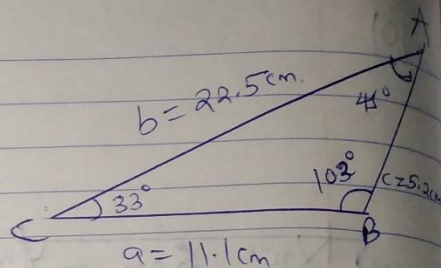
$$\sin B = \frac{b \sin C}{c} = \frac{22.5 \sin 33^\circ}{5.2} = 2.325^+$$

$$\angle A = 45^\circ$$

The triangle formed has sides

$$a = 11.1 \text{ cm}, b = 22.5 \text{ cm}, c = 5.2 \text{ cm}$$

$$\angle A = 45^\circ, \angle B = 103^\circ, \angle C = 33^\circ$$



(7)

$$\text{Period} = 20$$

$$\text{Range } (y \in \mathbb{R})$$

$$3 \leq y \leq 8$$

Graph on the maths page

$$(8) (a) y = 5 \cos(2x) + 7$$

Sketched on grapher

$$b) y = -0.5 \sin(x - 30) - 4$$

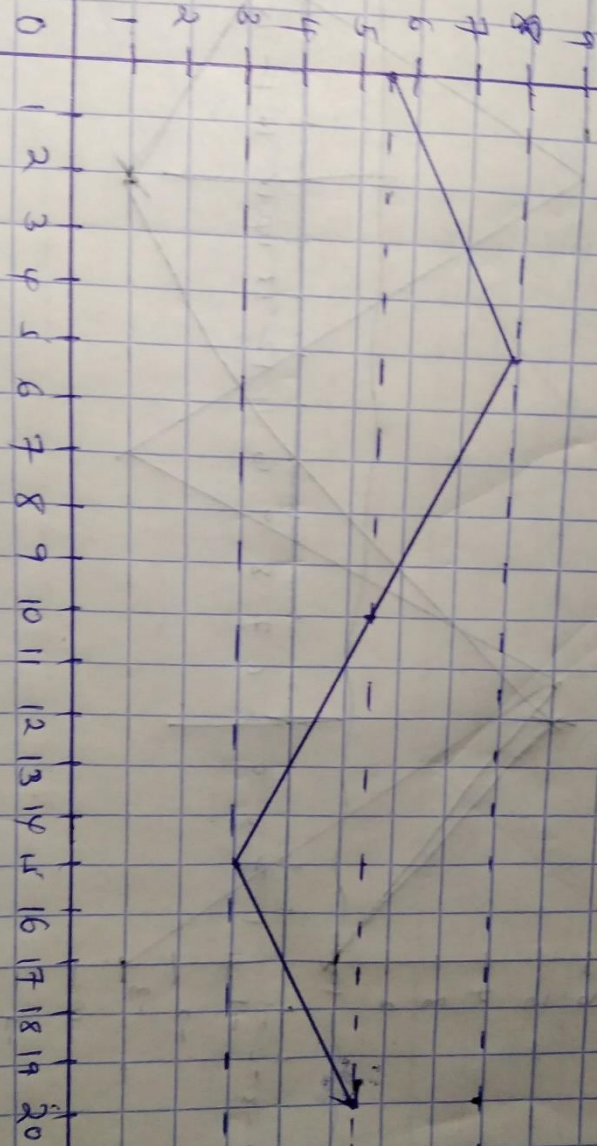
$$(c) y = -\cos\left(\frac{1}{3}x\right) + 1$$

$$(d) y = \frac{1}{4} \sin x + 0.5$$

$$(9) (a) y = 2 \cos(360x + 720) - 3$$

Period 20

Range  $3 \leq y \leq 8$



constant



# Graph



3D



$$5\cos(2x)+7$$

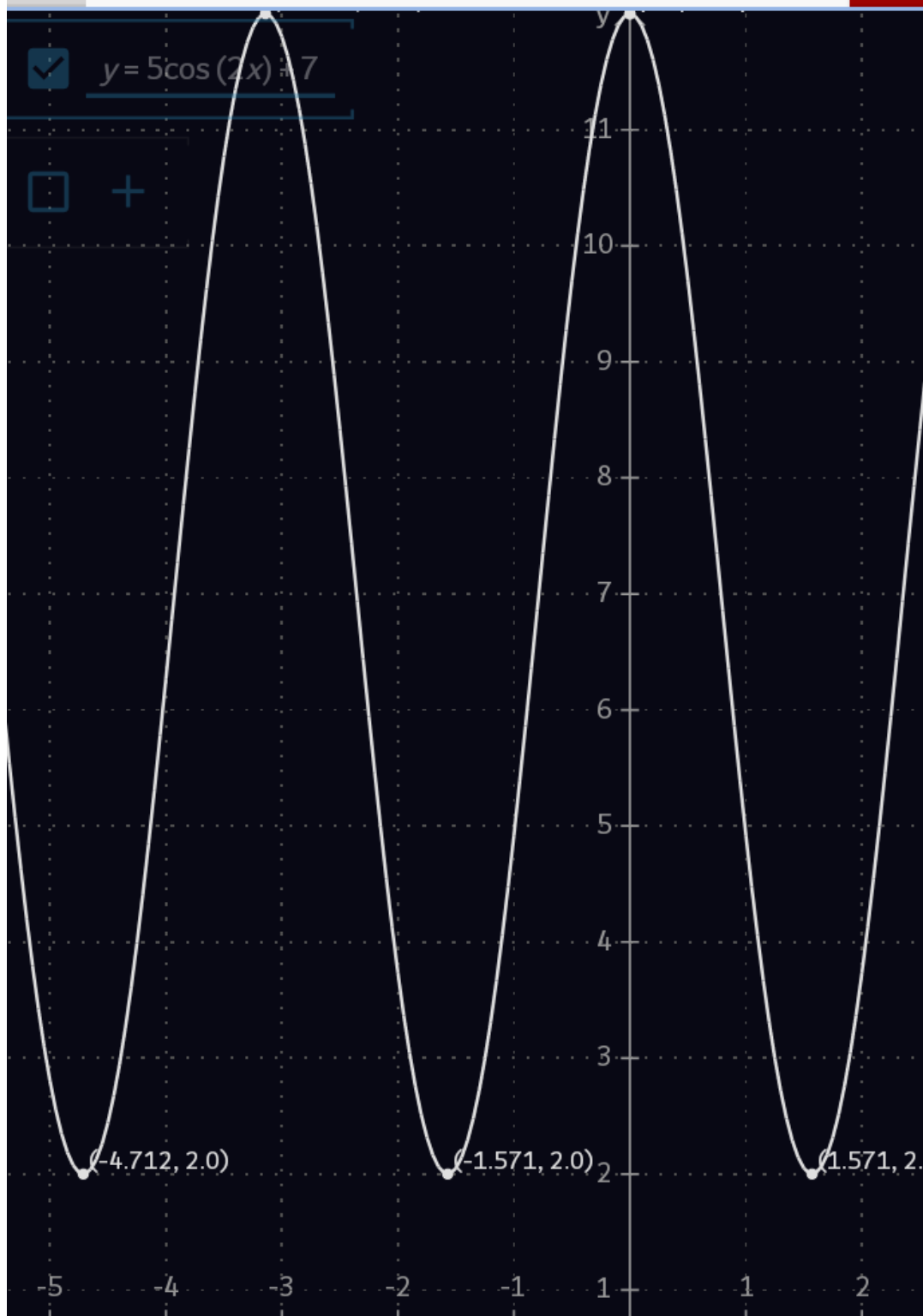
C



$$y = 5\cos(2x) + 7$$



+



1

2

3

0-9

a-z

α-ω



# Graph

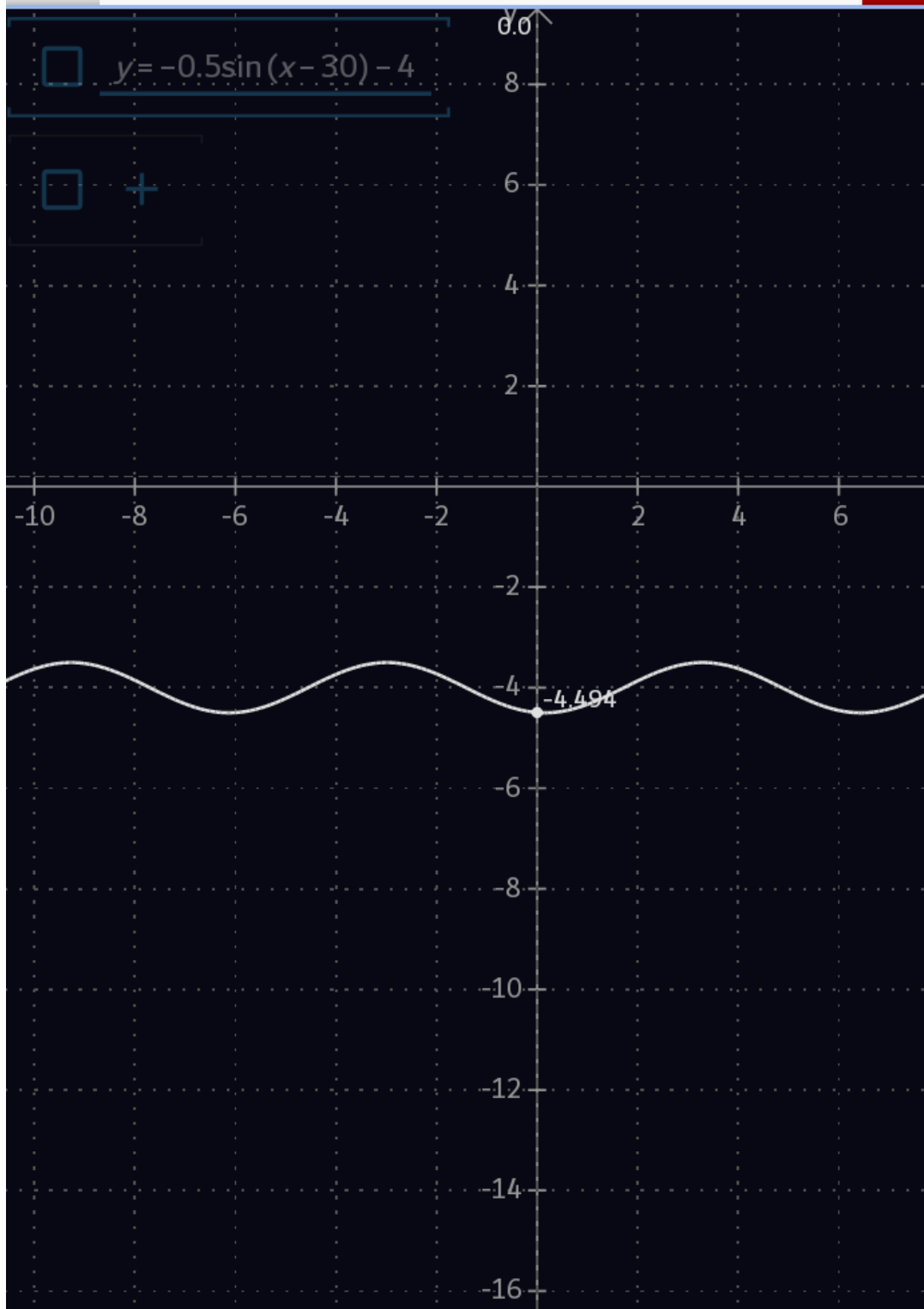


3D



$-0.5\sin(x-30)-4$

C



1

2

3

0-9

a-z

a-ω



# Graph



3D



$$-\cos(1/3x)+1$$

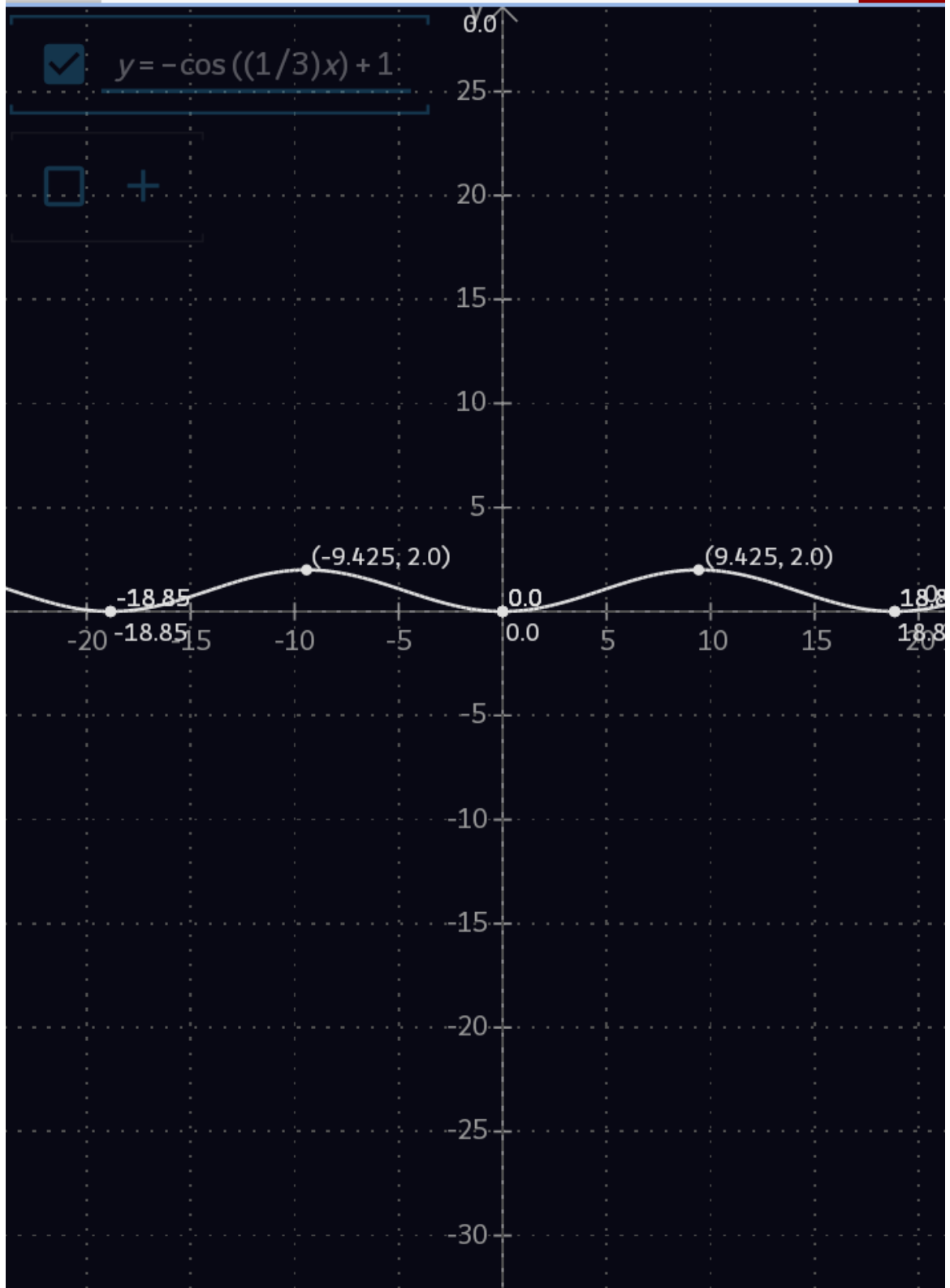
C



$$y = -\cos((1/3)x) + 1$$



+





# Graph



3D



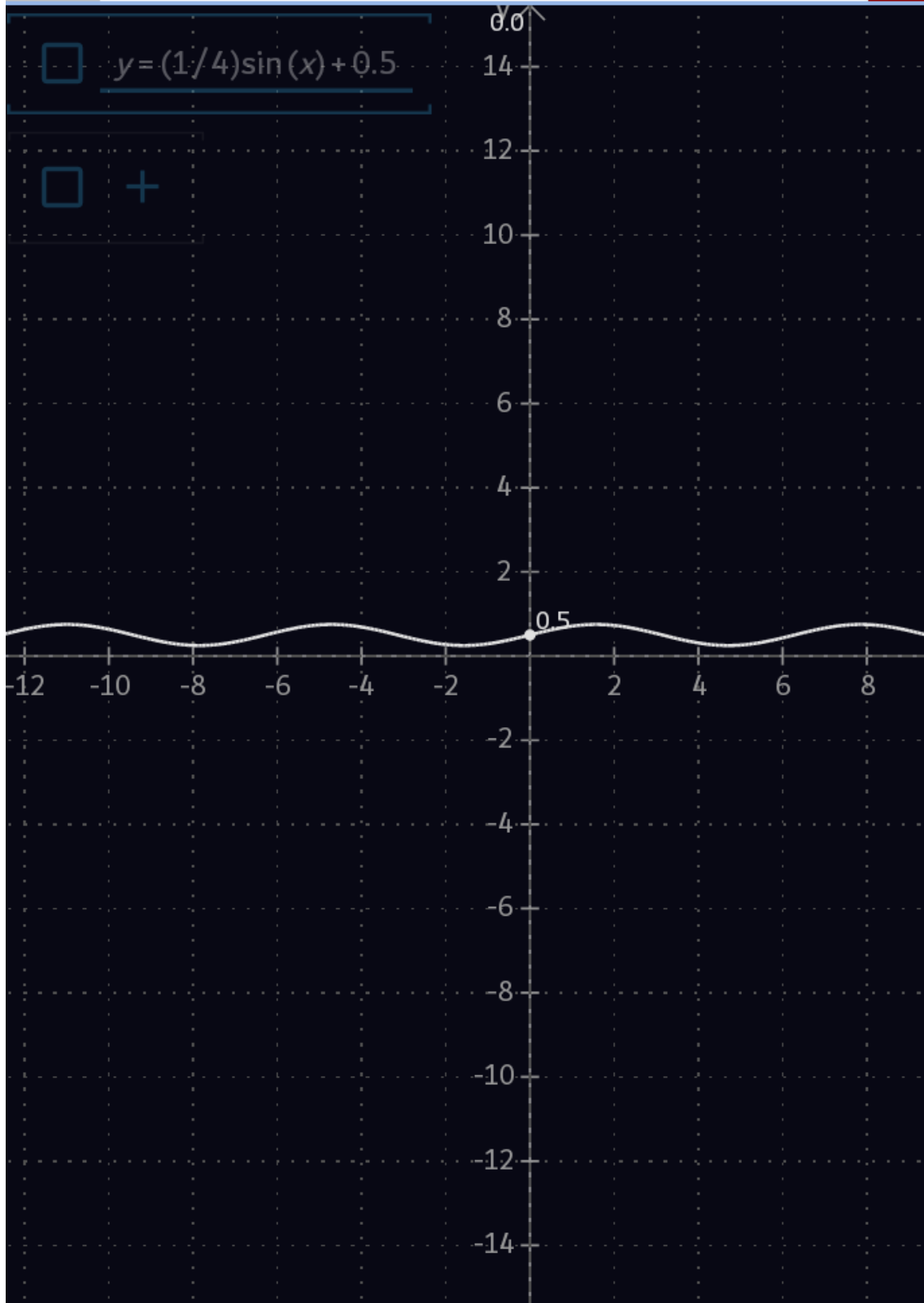
$$1/4\sin(x)+0.5$$

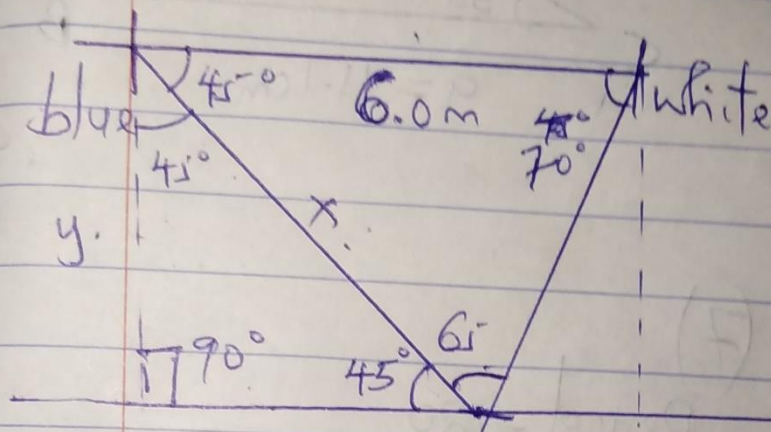


$$y = (1/4)\sin(x) + 0.5$$



+





Using Sine Rule

$$\frac{6}{\sin 65^\circ} = \frac{x}{\sin 70^\circ}$$

$$x = \frac{6 \sin 70^\circ}{\sin 65^\circ} = \underline{\underline{6.22 \text{ m}}}$$

$$\frac{6.22}{\sin 90^\circ} = \frac{y}{\sin 45^\circ}$$

$$y = \frac{6.22 \sin 45^\circ}{\sin 90^\circ}$$

$$= \underline{\underline{4.398 \text{ m}}}$$

$$= \underline{\underline{4.4 \text{ m}}}$$

9a)  $y = 2 \cos(360x + 720) - 3$

The general equation:  $y = a \cos(bx - c) + d$

Phase shift:  $-\frac{c}{b} = -\frac{720}{360} = -2$  (2 to the left)

Vertical shift:  $d = -3$

Amplitude:  $|a| = |2| = 2$

Period:  $\frac{2\pi}{b} = \frac{2\pi}{360} = \frac{\pi}{180} = 1$

Domain:  $(-\infty, \infty)$

Range:  $[d - |a|, d + |a|] \Rightarrow [-5, -1] \Rightarrow (y \in \mathbb{R} \mid -5 \leq y \leq -1)$

Max:  $-1$

Min:  $-5$

Equation of the axis:  $\frac{-5 + (-1)}{2} \Rightarrow y = -3$

b)  $y = \frac{3}{4} \sin(2(x + 30)) + 2 \Rightarrow y = \frac{3}{4} \sin(2x + 60) + 2$

Vertical shift:  $d = 2$

Phase shift:  $-\frac{c}{b} = -\frac{60}{2} = -30$  (30 units to the left)

Amplitude:  $|a| = \left|\frac{3}{4}\right| = \frac{3}{4}$

Period:  $\frac{2\pi}{b} = \frac{2\pi}{2} = \pi$

Domain:  $(-\infty, \infty)$

Range:  $\{y \in \mathbb{R} \mid \frac{7}{4} \leq y \leq \frac{15}{4}\}$

Equation of the axis:  $\frac{\frac{7}{4} + \frac{15}{4}}{2} \Rightarrow y = \frac{11}{4}$

Max:  $\frac{11}{4}$

Min:  $\frac{5}{4}$

c)  $y = -3 \cos(-x+1) + 1$

Phase shift:  $-\frac{c}{b} = \frac{-1}{-1} = 1$  (1 unit to the right)

Vertical shift:  $d = 1$

Amplitude:  $|a| = |-3| = 3$

Period:  $\frac{2\pi}{b} = \frac{2\pi}{1} = 2\pi$

Domain:  $(-\infty < x < \infty)$

Range:  $(-2 \leq y \leq 4 \mid y \in \mathbb{R})$

Max: 4

Min: -2

Equation of the axis:  $\frac{4 + (-2)}{2} = 1 \Rightarrow y = 1$

d)  $y = \frac{1}{2} \sin\left(\frac{2}{3}x\right)$

Phase shift: 0

Vertical shift: 0

Amplitude:  $|a| = \left|\frac{1}{2}\right| = \frac{1}{2}$

Period:  $\frac{2\pi}{b} = \frac{2\pi}{\frac{2}{3}} = 3\pi$

Domain:  $(-\infty < x < \infty)$

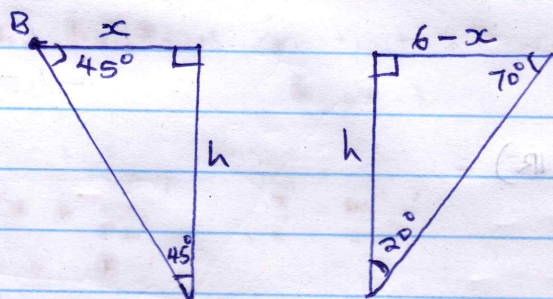
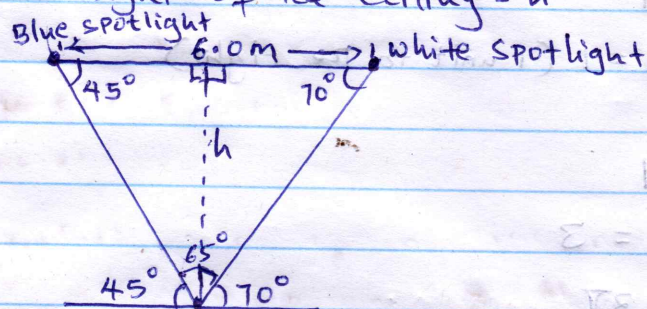
Range:  $\{y \in \mathbb{R} \mid -\frac{1}{2} \leq y \leq \frac{1}{2}\}$

Max:  $\frac{1}{2}$

Min:  $-\frac{1}{2}$

Equation of the axis:  $y = \frac{1}{2} - \frac{1}{2} = 0 \Rightarrow y = 0$

10. Let the height of the ceiling =  $h$



$$\Rightarrow \tan 45 = \frac{x}{h} \quad \text{and} \quad \tan 20 = \frac{6-x}{h}$$

$$h = \frac{x}{\tan 45}, \quad h = \frac{6-x}{\tan 20}$$

$$\frac{x}{\tan 45} = \frac{6-x}{\tan 20}$$

$$x \tan 20 = (6-x) \tan 45$$

$$x \tan 20 = 6 \tan 45 - x \tan 45$$

$$x \tan 20 + x \tan 45 = 6 \tan 45$$

$$x(\tan 20 + \tan 45) = 6 \tan 45$$

$$x = \frac{6 \tan 45}{\tan 20 + \tan 45}$$

$$h = \frac{x}{\tan 45}$$

$$h = \left( \frac{6 \tan 45}{\tan 20 + \tan 45} \right) \frac{1}{\tan 45} = \frac{6}{\tan 20 + \tan 45} = 4.4 \text{ m}$$

ii.  $h(t) = 25 \sin(30(t-30)) + 27$

The general form:  $h(t) = a \sin(b(t-c)) + d$

midline:  $d = 27$

period =  $\frac{2\pi}{15}$

Amplitude:  $a = 25$

Angular speed:  $b = 30$

$h(\theta) = 25 \sin(\theta - 90) + 27$

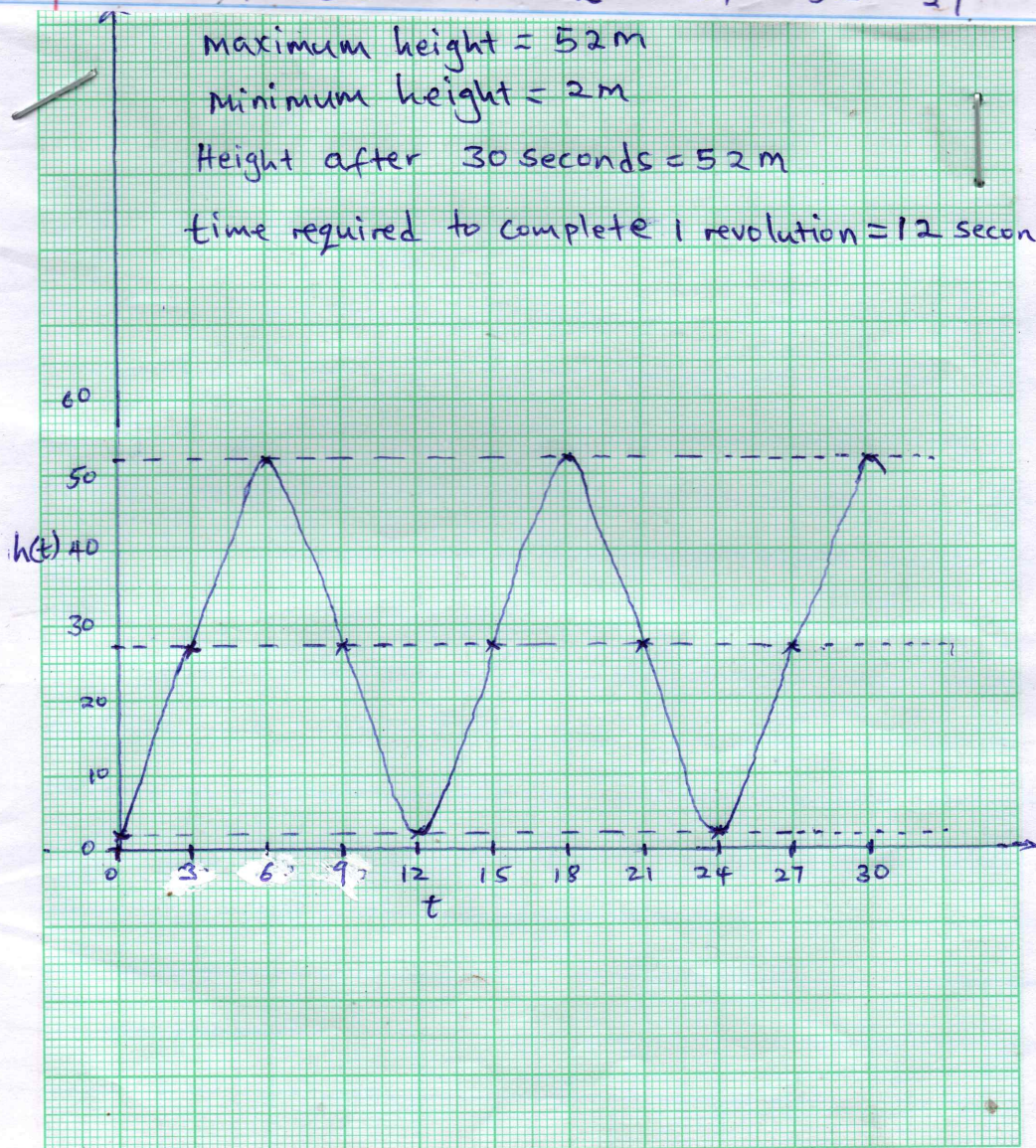
|          |   |    |     |     |     |     |     |     |     |     |     |
|----------|---|----|-----|-----|-----|-----|-----|-----|-----|-----|-----|
| $\theta$ | 0 | 90 | 180 | 270 | 360 | 450 | 540 | 630 | 720 | 810 | 900 |
| $t$      | 0 | 3  | 6   | 9   | 12  | 15  | 18  | 21  | 24  | 27  | 30  |
| $h$      | 2 | 27 | 52  | 27  | 2   | 27  | 52  | 27  | 2   | 27  | 52  |

Maximum height = 52m

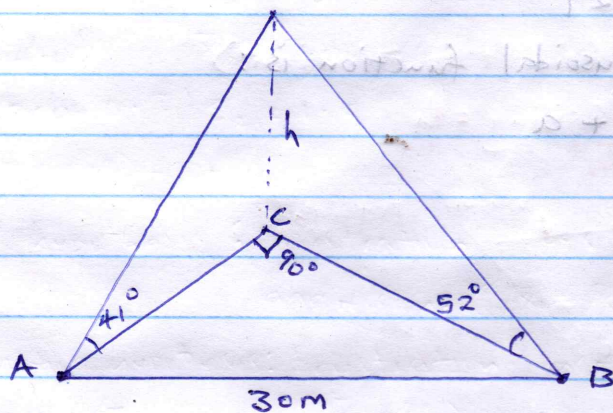
Minimum height = 2m

Height after 30 seconds = 52m

time required to complete 1 revolution = 12 seconds.



12.



$$\tan 41^\circ = \frac{h}{AC} \Rightarrow AC = \frac{h}{\tan 41^\circ}$$

$$\tan 52^\circ = \frac{h}{BC} \Rightarrow BC = \frac{h}{\tan 52^\circ}$$

$$(AB)^2 = (AC)^2 + (BC)^2$$

$$30^2 = \left( \frac{h}{\tan 41^\circ} \right)^2 + \left( \frac{h}{\tan 52^\circ} \right)^2$$

$$30^2 = h^2 \left( \frac{1}{\tan 41^\circ} + \frac{1}{\tan 52^\circ} \right)$$

$$h^2 = 30^2 / \left( \frac{1}{\tan 41^\circ} + \frac{1}{\tan 52^\circ} \right)$$

$$h^2 = 465.9219$$

$$h = 22m$$

13. Amplitude = 2

Period = 180

max at (0, 3)

$$y = a \sin(b(x-c)) + d$$

$a = 2$

Minimum value = -1

midline is at  $y = 1$

$$b = \frac{2\pi}{\frac{180}{360}} = 2$$

$$y = 2 \sin(2(x-c)) + 1 \Rightarrow y = 2 \sin(2x + \frac{\pi}{2}) + 1$$

and

$$y = 2 \cos(2(x-c)) + 1 \Rightarrow y = 2 \cos(2x) + 1$$

14.

a)  $80 - 0 = 80$  seconds

The period = 80 represent the time it takes the ferris wheel to make one complete revolution.

b) Equation of the axis:  $h = 16$  m

$h = 16$  m represents the distance between the ground and the center of the ferris wheel.

c) Amplitude = 9

The Amplitude represents the radius of the ferris wheel

d)  $s = \frac{\pi D}{\text{Period}} = \frac{18\pi}{80} = 0.71 \text{ m s}^{-1}$

e)  $7 - 6 = 1$  m